Heavy Quark Expansion for Inclusive Semileptonic Charm Decays Revisited

Keri Vos

in collaboration with M. Fael and Th. Mannel

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How to handle the charm mass?

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Can we apply HQE to charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Lifetimes $\rightarrow$ Stay tuned for Alex
- Inclusive semileptonic decays CLEO coll. PRD 81 (2010) 052007

$$\Gamma(D^+ \to Xe^+\nu_e)/\Gamma(D^0 \to Xe^+\nu_e) = 0.985 \pm 0.015 \pm 0.024$$
$$\Gamma(D_s^+ \to Xe^+\nu_e)/\Gamma(D^0 \to Xe^+\nu_e) = 0.828 \pm 0.051 \pm 0.025$$

- Weak Annihilation effects $\mathcal{O}(\Lambda^3_{\rm QCD}/m_c^3)$ found to be small Ligeti, Luke, Manohar, hep-ph/1003.1351; Gambino, Kamenik, hep-ph/1004.0114

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Why HQE for charm?

- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

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Challenges:

- Valance and non-valance WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

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In short: how to handle the charm mass?

The HQE for charm

I: $m_Q \sim m_q \gg \Lambda_{\rm QCD}$ OPE for $b \to c\ell\bar{\nu}$

- q is treated as a heavy degree of freedom
- two-quarks operators
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \ldots\right]\frac{\rho_D}{m_b^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

II: $m_Q \gg m_q \gg \Lambda_{\rm QCD}$ start with q dynamcial

- four-quark operators $(\bar{Q}_v\Gamma q)(q\bar{\Gamma}Q_v)$

$\to$ removed when matching onto two-quark ops

- RGE running gives $\log(m_q/m_Q)$

III: $m_Q \gg m_q \sim \Lambda_{\rm QCD}$ OPE for $c \to s\ell\bar{\nu}$

- q dynamical degree of freedom
- four-quark operators remain in OPE
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IV: $m_Q \gg \Lambda_{\rm QCD} \gg m_q$ for $b \to u$ and $c \to d$ transitions

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Inclusive $b \to c$ vs $c \to s$

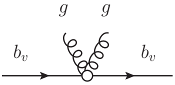
$\boldsymbol{m_b}$

$\boldsymbol{m_c}$

$$\sum_{n,i} \frac{C_{i,n}(\frac{m_c}{m_b})}{m_b^n} \langle B|O^{2q}|B\rangle$$

$\Lambda_{\rm QCD}$

$C_2 \times$



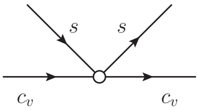
$\boldsymbol{m_c}$

$$\sum_{n,i} \frac{C_{i,n}(\frac{\mu}{m_c})}{m_c^n} \langle D|O^{2q}|D\rangle$$

$$+\frac{C_{j,n}}{m_c^n} \langle D|O^{4q}|D\rangle$$

$\Lambda_{\rm QCD}$

$\boldsymbol{m_s}$



HQE for Charm revisited

- The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

- Expansion parameters:
  - $1/m_c$
  - α_s
  - m_s/m_c

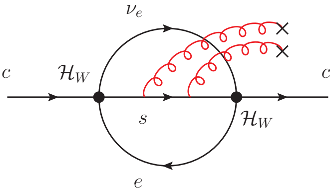
Fael, Mannel, KKV, hep-ph/1910.05234

HQE for Charm revisited

- Systematic treatment of four-quark operators order by order in $1/m_Q$
- Set up OPE directly for $\Gamma_{\rm tot}$ and $\langle M^{(n)} \rangle$

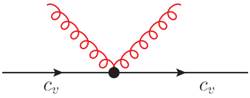
  following the idea in Bauer, Falk, Luke hep-ph/9604290

$2\,\mathrm{Im}$

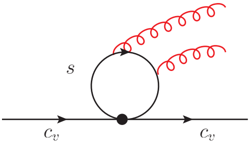


$\longrightarrow$

$C^{2\mathrm{q}}(\mu) \times$



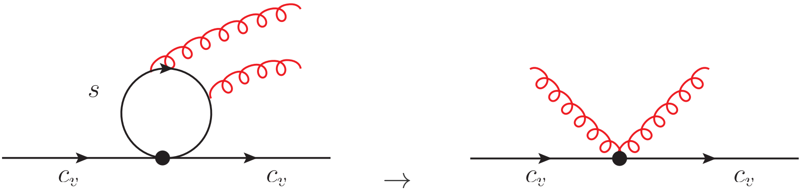
$+\, C^{4\mathrm{q}}(\mu) \times$



HQE for Charm revisited

- $\log(m_c/m_b)$ in $B \to X\ell\nu$ corresponds to $\log(\mu/m_c)$ in $D \to X\ell\nu$
- caused by mixing of four-quark operators into two-quark operators:

$$C_i^{2q}(\mu) = C_i^{2q}(m_c) + \log\left(\frac{\mu}{m_c}\right)\sum_j \hat{\gamma}_{ij}^T C_j^{4q}(m_c)$$



HQE for Charm revisited

Fael, Mannel, KKV

- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{4\mathrm{q}}|D\rangle$
- Up to $1/m_c^3$ only <u>one</u> extra HQE param:

$$\tau_0 = 128\pi^2 \left(T_1(\mu) - T_2(\mu) - 2\frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c} \right)$$
$$+ \log\left(\frac{\mu^2}{m_c^2}\right) \left[8\tilde{\rho}_D^3 + \frac{1}{m_c}\left(\frac{16}{3} r_G^4 - \frac{16}{3} r_E^4 + \frac{8}{3} s_E^4 - \frac{1}{3} s_{qB}^4 - 12 m_s^4 \right) \right]$$

- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

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Key question: HQE is indeed applicable to inclusive charm decays?

HQE for charm revisited

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Key question: How to hadle the charm mass?

How to handle the charm mass?

Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- $\overline{\text{MS}}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\text{kin}} = m_Q^{\text{Pole}} - \left[\overline{\Lambda}\right]_{\text{pert}} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\text{pert}} + \ldots$$

$$[\overline{\Lambda}]_{\text{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\text{pert}} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher order term in the HQE generates corrections $(\alpha/\pi)\mu^n/m_Q^n$.
- $\Lambda_{\text{QCD}} < \mu < m_Q$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??

    $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$
    $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

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Kinetic Mass

Putting all power corrections to zero!

- $m_c^{\rm kin}(1\text{ GeV}) = 1.16\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 7.7\frac{\alpha_s(m_c)}{\pi} + 69\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

- $m_c^{\rm kin}(0.5\text{ GeV}) = 1.4\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 1.2\frac{\alpha_s(m_c)}{\pi} + 17\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

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$\mu = 0.5$ GeV touches upon the non-perturbative regime?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0|T[j_\mu(x) j_\nu(0)]|0\rangle = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)$$

- Expand around $q^2 = 0$: ($\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots$)

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty} \bar{C}_n \left(\frac{q^2}{4m_c^2}\right)$$

- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int \frac{ds}{s^{n+1}} R(s) \qquad (1)$$

- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

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Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0| T[j_\mu(x) j_\nu(0)]|0\rangle = (g_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty} \bar{C}_n \left(\frac{q^2}{4m_c^2}\right) = \Pi(0) + \frac{q^2}{12\pi^2}\int \frac{ds}{s}\frac{R(s)}{s-q^2}$$

- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int \frac{ds}{s^{n+1}} R(s) \tag{1}$$

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Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

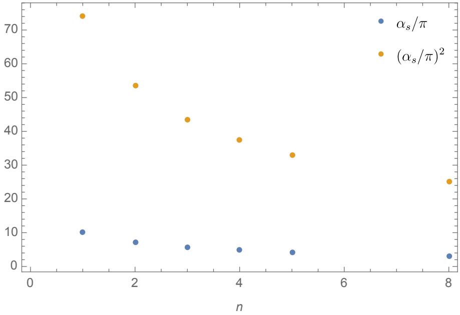
$$\Gamma(c \to s\ell\nu) = \frac{G_F^2 |V_{cs}|^2}{192\pi^3} \left(\frac{1}{2} \left(\frac{\bar{C}_n}{M_n} \right)^{1/2} \right)^5 \left(1 + \frac{\alpha_s(\mu)}{\pi} a_1 + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 a_2 + \cdots \right)$$

$$= \frac{G_F^2 |V_{cs}|^2}{6144\pi^3} \left(\frac{\bar{C}_n^{(0)}}{M_n} \right)^{5/2} \left(1 + \frac{\alpha_s(\mu)}{\pi} \left[a_1 + \frac{5}{2n} \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right] + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[a_2 + \frac{5}{2n} a_1 \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} + \frac{5}{2n} \frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}} + \frac{5}{4n} \left(\frac{5}{4n} - 1 \right) \left(\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right)^2 \right] + \cdots \right)$$

- $\frac{\bar{C}_n^{(1)}}{n\bar{C}_n^{(0)}}$: $5.1, 3.9, 3.3, 2.9, 2.7, x, x, 2.2$
- $\frac{\bar{C}_n^{(2)}}{n\bar{C}_n^{(0)}}$: $31.1, 35.5, 36.6, 37.1, 37.2, x, x, 37.2$

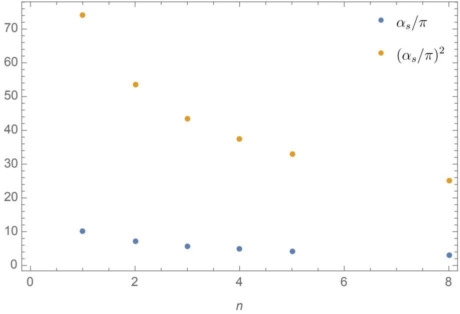
Spectral-Density Mass

PRELIMINARY!



Spectral-Density Mass

PRELIMINARY!



- How to use e^+e^- to extract the charm mass?
- Which scale to use for $\alpha_s(\mu)$?
- Other ideas?

Spectral-Density Mass

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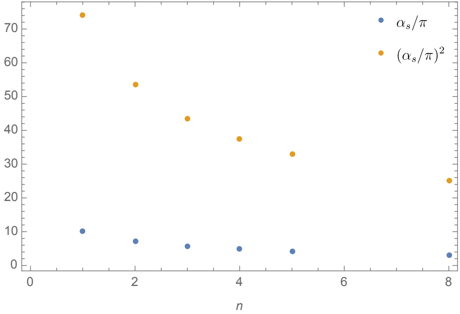

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Thanks and Lets discuss!

Outlook

Backup